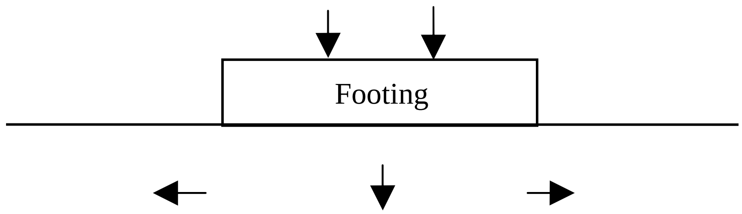
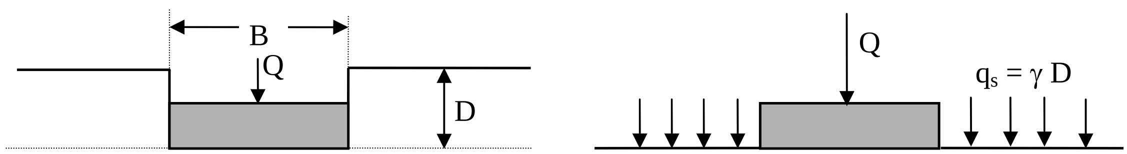
BEARING CAPACITY

If a footing is subjected to too great a load, some of the soil supporting it will reach a failure state and the footing may experience a bearing capacity failure. The bearing capacity is the limiting pressure that the footing can support.



Shallow Foundations (D/B < 1)

The solutions presented below have been determined for the situation where a footing sits on the soil surface. The effect of founding the footing below the ground surface, as is common for most buildings, is introduced by assuming that the soil above the footing applies a surcharge as shown below



Buried Footing Equivalent Surface Footing

In making this assumption the effects of any restraint from the soil above the base of the footing are ignored. This assumption is reasonable provided that the depth of burial is not too great. To be considered as a shallow foundation D/B should be less than 1.

There are two methods of theoretical analysis used to investigate the bearing capacity.

- The lower bound approach in which an arrangement of the stresses in the ground is found in equilibrium with the applied loads for which the soil is everywhere in a state of failure. The solution is less than or equal to the true collapse load.

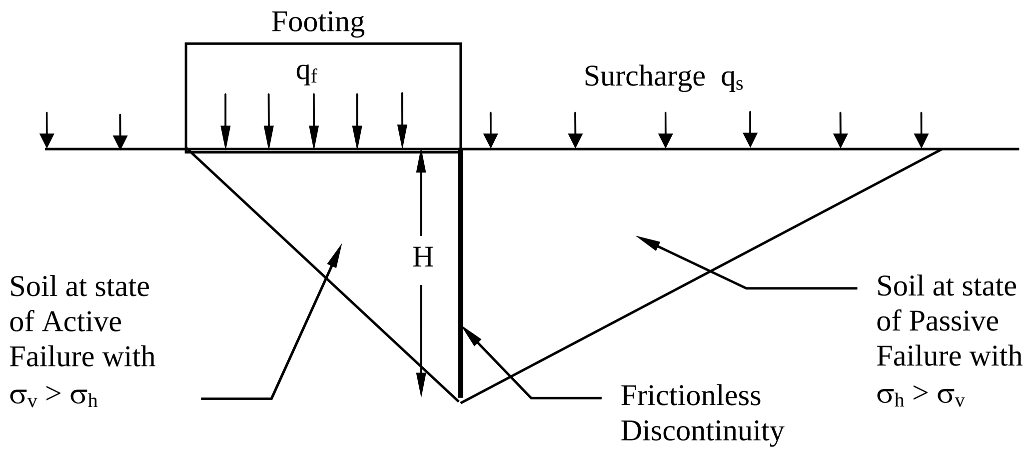
- The upper bound approach in which a failure mechanism is postulated and the applied loads in equilibrium with the assumed failure mechanism are determined. The solution is greater than or equal to the true collapse load.

- In the limit, when the mechanism and the stresses are consistent, the two methods give the same solution.

Accurate solutions to the collapse loads (bearing capacity) are difficult to determine when $\phi \neq 0$ and most of the charted solutions are not totally reliable. Computerised methods for determining bounds to the collapse loads are now available and are likely to be more widely used in the future.

Bearing capacity of a shallow strip footing (plane strain)

To illustrate the factors on which the bearing capacity depends a simple (not accurate) lower bound approach is considered below.



When the soil is at a state of failure the Mohr-Coulomb criterion is satisfied, and we have

$$\sigma_1 = N_\phi \sigma_3 + 2c\sqrt{N_\phi}$$

or

$$N_\phi = \frac{\sigma_1 + c\cot\phi}{\sigma_3 + c\cot\phi}$$

Underneath the footing the vertical stress will be greater than the horizontal stress, that is $\sigma_v = \sigma_1$ and $\sigma_h = \sigma_3$, and in the limit these will be related by the Mohr-Coulomb criterion. At any depth z the vertical stress is given by

$$\sigma_v = q_f + \gamma z$$

At failure

$$N_\phi = \frac{q_f + \gamma z + c\cot\phi}{\sigma_h + c\cot\phi}$$

hence

$$\sigma_h = \frac{1}{N_\phi}(q_f + \gamma z + c\cot\phi) - c\cot\phi$$

In the region away from the footing the horizontal stress will be greater than the vertical stress. At any depth z the vertical stress is given by

$$\sigma_v = q_f + \gamma z$$

and at failure

$$N_\phi = \frac{\sigma_h + c\cot\phi}{q_s + \gamma z + c\cot\phi}$$

and

$$\sigma_h = N_\phi(q_s + \gamma z + c\cot\phi) - c\cot\phi$$

For equilibrium the horizontal forces from the two failure zones must be equal

$$\int_0^H (\sigma_h)_{active}\, dz \;=\; \int_0^H (\sigma_h)_{passive}\, dz$$

which gives

$$\frac{1}{N_\phi}\left[q_f\, H + \frac{\gamma\, H^2}{2} + c \cot\phi\, H\right] \;=\; N_\phi\left[q_s\, H + \frac{\gamma\, H^2}{2} + c \cot\phi\, H\right]$$

and hence the bearing capacity q_f is

$$q_f \;=\; q_s\, N_\phi^2 \;+\; \frac{\gamma\, H}{2}\left(N_\phi^2 - 1\right) \;+\; c\cot\phi\left(N_\phi^2 - 1\right)$$

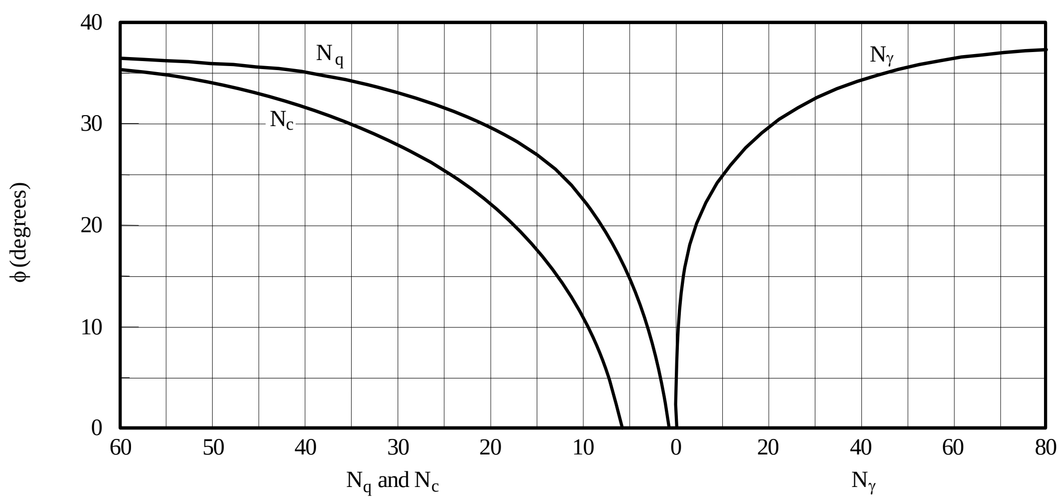
This solution will give a lower bound to the true solution because of the simplified stress distribution assumed in the soil. Nevertheless, similar terms occur in all bearing capacity calculations. These terms relate to

- the surcharge applied to the soil surface
- the self weight of the soil
- the effect of cohesion

The general bearing capacity formulae are therefore written in the following form

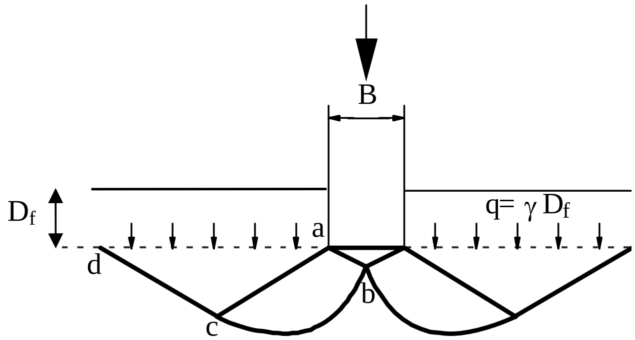
$$q_f \;=\; q_s\, N_q \;+\; \frac{\gamma\, B}{2} N_\gamma \;+\; c\, N_c$$

where the quantities N_q, N_γ, and N_c are known as bearing capacity factors. For the shallow bearing capacity problem values of these factors depend on ϕ and can be determined from the chart produced by Terzaghi shown below (see also p 28 in Data Sheets)



BEARING CAPACITY FACTORS [After Terzaghi and Peck (1948)]

The mechanism analysed by Terzaghi is shown below. Note that it differs from the simple problem analysed above by having a transition region between the regions where the principal stresses are horizontal and vertical.



The shape of the footing also influences the bearing capacity. This can be allowed for by adjusting the terms in the bearing capacity equation.

For a continuous strip footing $$q_f = q_s N_q + \frac{\gamma B}{2} N_\gamma + c N_c$$

For a square footing $$q_f = q_s N_q + 0.4 \gamma B N_\gamma + 1.3 c N_c$$

For a circular footing $$q_f = q_s N_q + 0.6 \gamma B N_\gamma + 1.3 c N_c$$

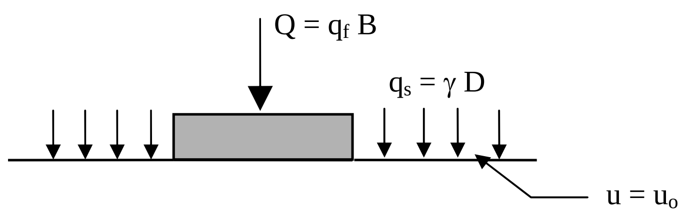
The analysis has been presented in terms of total stress. This can be used to evaluate the short term undrained bearing capacity. To evaluate the long term bearing capacity an effective stress analysis is required. This is very similar to the total stress analysis considered above.

Effective Stress Analysis

Two situations can be simply analysed.

- The soil is dry. The total and effective stresses are identical and the analysis is identical to that described above except that the parameters used in the equations are c', ϕ', γ_{dry} rather than c_u, ϕ_u, γ_{sat}. If the water table is more than a depth of 1.5 B (the footing width) below the base of the footing the water can be assumed to have no effect.

- The soil below the base of the footing is saturated.



Notation

The effective bearing capacity $q'_f = q_f - u_o$

The effective surcharge $q'_s = q_s - u_o$

The effective (submerged) unit weight $\gamma' = \gamma_{sat} - \gamma_w$

These quantities are used because they give the variation of effective stress with depth. For example under the footing the total vertical stress, pore pressure and effective vertical stress at any depth z are

$$\sigma_v = q_f + \gamma z$$

$$u = u_o + \gamma_w z$$

$$\sigma'_v = \sigma_v - u = q'_f + \gamma' z$$

Following through the same analysis as before but using the effective stress failure criterion which is given by

$$N_\phi = \frac{\sigma'_1 + c' \cot\phi'}{\sigma'_3 + c' \cot\phi'}$$

gives

$$q'_f = q'_s N_\phi^2 + \frac{\gamma' H}{2}\left(N_\phi^2 - 1\right) + c' \cot\phi' \left(N_\phi^2 - 1\right)$$

which can be more generally written as

$$q'_f = q'_s N_q + \frac{\gamma' B}{2} N_\gamma + c' N_c$$

where the bearing capacity factors are identical to those given before. Note that the total bearing capacity is q_f and

$$q_f = q'_f + u_o$$

The bearing capacity is a function of many factors. The simple analysis considered so far has accounted for

- soil strength parameters
- rate of loading (drained or undrained)
- groundwater conditions (dry or saturated)
- foundation shape (strip footing, square or circle)

Additional factors which also influence the capacity are

- soil compressibility
- embedment (D/B > 1)
- inclined loading
- eccentric loading
- non-homogeneous soil

These are usually taken into account by introducing correction factors to the conventional bearing capacity factors. Pages 74 and 75 of the Data Sheets give formulae for some of these correction factors.

It should also be noted that pages 69 to 71 of the Data Sheets give more (theoretically) accurate bearing capacity factors. However, in practice the Terzaghi factors are still widely used. There are two important points to note about these sets of factors

- The bearing capacity equation assumes that the effects of c', γ, and ϕ' can be superimposed. However, this is not correct as there is an interaction between the three effects. This is a consequence of the plastic nature of the soil response.

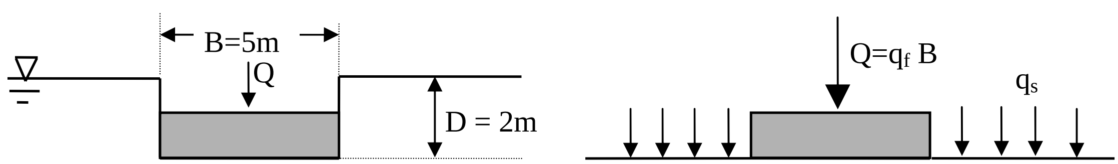
- The formulae give the ultimate bearing capacity. In practice the soil will deform significantly before general bearing failure occurs and large settlements may occur. Local failure (yield) will occur at some depth beneath the footing at a load less than the ultimate collapse load. The zone of plastic (yielding) soil will then spread as the load is increased. Only when the failure zone extends to the surface will a failure mechanism exist.

A minimum load factor of 3 against ultimate failure is usually adopted to keep settlements within acceptable bounds, and to avoid problems with local failure.

Example

A 5 m wide strip footing is constructed on saturated clay with properties c_u = 25 kN/m^2, ϕ_u = 0, c' = 2 kN/m^2, ϕ' = 25^o, and γ_{sat} = 15 kN/m^2.

Determine the short term and long term bearing capacities if the water table is at the soil surface and the footing is founded 2 m below the surface. The figure below shows the actual and idealised problem to be analysed.



1. Short term - Undrained analysis

The position of the water table is not important, but the soil must be saturated for an undrained analysis to be appropriate.

$q_s = \gamma_{sat} D = 15 \times 2 = 30$ kPa

For $\phi_u = 0$ $N_\gamma = 0$, $N_q = 1$, $N_c = 5.14$ (From Terzaghi's chart)

For $\phi_u = 0$ values can be more accurately read from Skempton's chart also on p28 in Data Sheets, and this is discussed further below.

$$q_f = q_s N_q + \frac{\gamma B}{2} N_\gamma + c N_c$$

$q_f = 30 \times 1 + 0 + 25 \times 5.14 = 158.5$ kPa   (Bearing capacity)

$Q = q_f \times B = 158.5 \times 5 = 792.5$ kN/m (Bearing Force)

2. Long term - Effective stress analysis

$q_s = 30$ kPa
$u_o = 2 \times 9.8 = 19.6$ kPa
$q'_s = 10.4$ kPa
$\gamma' = 15 - 9.8 = 5.2$ kPa

For $\phi' = 25^o$ $N_q = 13$, $N_c = 24.5$, $N_\gamma = 10$ (From Terzaghi's chart)

$$q'_f = q'_s N_q + \frac{\gamma' B}{2} N_\gamma + c' N_c$$

$q'_f = 10.4 \times 13 + 0.5 \times 5.2 \times 5 \times 10 + 2 \times 24.5 = 314.2$ kPa

$q_f = 314.2 + 19.6 = 333.8$ kPa

$Q = 1669$ kN/m

Total stress analysis for $\phi_u = 0$

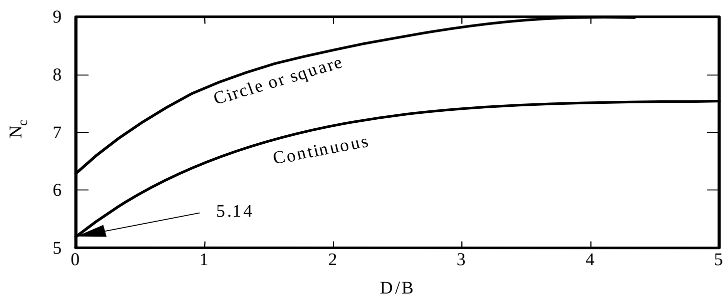
The analysis is more straightforward for $\phi_u = 0$ soil. The bearing capacity formula reduces to

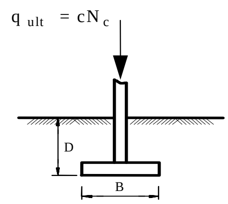
$$q_f = N_c \ c_u + q_s$$

Values of N_c can be obtained from the chart produced by Skempton shown below (p28 in Data Sheets). It can be seen that N_c varies with the depth to width ratio D/B and with the shape of the footing. This chart is also applicable to deep foundations, that is with D/B > 1.

It is often assumed that the stress due to the weight of the footing, and any soil used to bury the footing, is equivalent to the soil stress q_s and thus the net bearing capacity can be written

$$q_{ult} = c_u N_c$$





N_c (for rectangle)

$= (0.84+0.16\frac{B}{L})\, N_c$ (square)

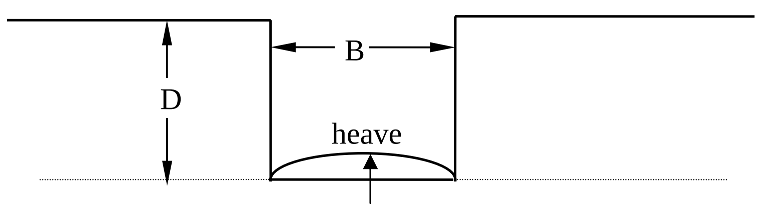
L= Length of footing

ULTIMATE BEARING CAPACITY OF CLAY (ϕ = 0 only) (After A.W. Skempton)

$$q_f = cN_c + \gamma D$$

Bottom heave of excavations in clay

During the excavation of well supported trenches and larger holes in the ground it is possible for failure to occur as the soil heaves up into the base of the excavation. This mode of failure is a kind of bearing capacity failure. The weight of clay besides the excavation tends to push the underlying clay into the excavation.



For $\phi = 0$, and constant undrained strength c_u

The bearing capacity (pressure) $= c_u N_c$

The driving pressure causing failure $= \gamma D$

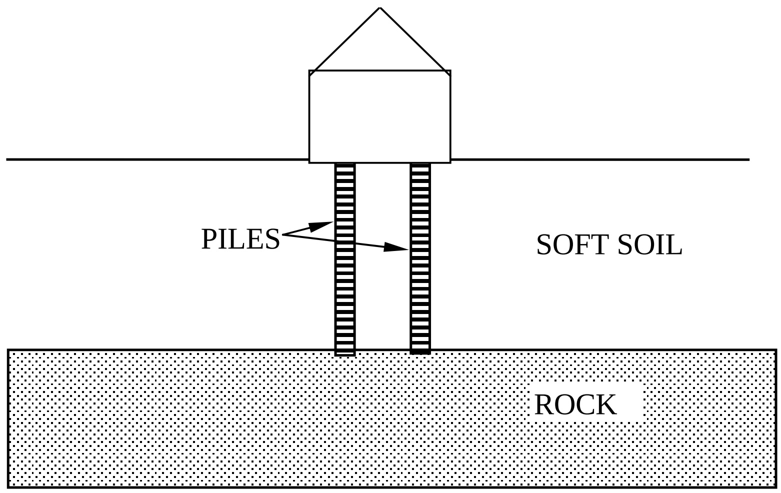
and the Factor of Safety $= \dfrac{\text{Bearing capacity}}{\text{Stress causing failure}} = \dfrac{c_u N_c}{\gamma D}$

Values for N_c can be determined from Skempton's chart given above. The restraining effects of the soil around the excavation have been ignored. For shallow excavations this has the effect of slightly increasing the factor of safety.

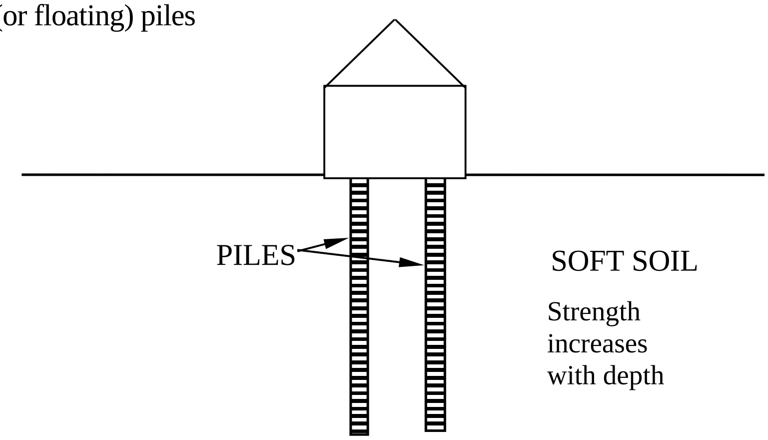
Deep (Pile) foundations

Piles are relatively long and slender members used to transmit foundation loads through soil strata of low bearing capacity to deeper soil or rock having a higher bearing capacity. The method by which this occurs is the basis of the simplest pile type classification. We have two main pile types:

1. End-bearing piles



2. Friction (or floating) piles



For both pile types a further distinction is required based on their method of installation.

a. Driven (or displacement) piles: These piles are generally pre-formed before being driven, jacked, screwed or hammered into the ground.

b. Bored piles: For these piles a hole is first bored in the ground, and the pile is then usually formed in the hole.

These categories may be further subdivided into

Large Displacement

- Preformed - driven into the ground and left in position

  - Solid - Timber/Concrete
  - Hollow with a closed end - Steel or concrete tubes

- Formed in-situ - closed-ended tubular driven then withdrawn filling void with concrete

Small displacement

- Screw piles
- Steel tube and H-sections - (Tube sections may plug and become large displacement)

No displacement

- Void formed by boring or excavation then filled with concrete. During construction the hole may need to be supported for which there are two main options

  - steel casing
  - drilling mud

Loads applied to Piles

Combinations of vertical, horizontal and moment loading may be applied at the soil surface from the overlying structure.

For the majority of foundations the loads applied to the piles are primarily vertical. Horizontal loads arising from wind loads on structures are usually relatively small and are ignored.

However, for piles in jetties, foundations for bridge piers, tall chimneys, and offshore piled foundations the lateral resistance is an important consideration.

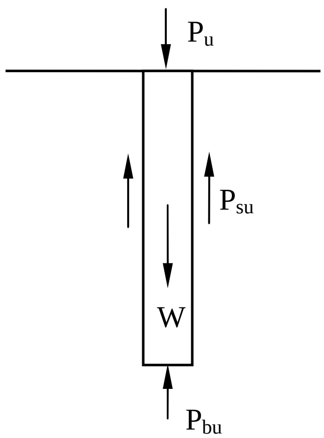
Only the analysis of piles subjected to vertical loads is considered here. The analysis of piles subjected to lateral and moment loading is more complex because of the nature of the soil-structure interaction.

Apart from their ability to transmit foundation loads to underlying strata piles are also widely used as a means of controlling settlement and differential settlement. In these notes only the ultimate axial capacity is considered.

Vertically Loaded Piles

Ultimate Capacity of Single Piles

The total pile resistance may be split into components from the base and the shaft. Consideration of static equilibrium then gives the ultimate capacity as:



$$P_u = P_{su} + P_{bu} - W$$

P_u = Ultimate load capacity of the pile

P_{bu} = Ultimate resistance at the base of the pile (Base resistance)

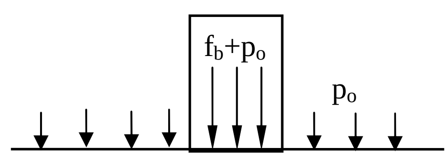
P_{su} = Ultimate side shear resistance on the pile shaft (Shaft resistance)

W = Self-weight of the pile

Base Resistance

In analysing pile behaviour it is conventional to express the ultimate base resistance by

$$P_{bu} = A_b\,(f_b + p_o)$$



A_b = Plan area of the pile base

f_b = Net ultimate resistance per unit area of the base

p_o = Overburden pressure at the level of the base

If the pile does not extend above the soil surface it is found that the pile weight is usually similar to the force due to the overburden pressure. Thus

$$W \approx A_b\,p_o$$

and

$$P_u = P_{su} + A_b\,f_b$$

Side Resistance

$$P_{su} = \bar{f}_s A_s$$

A_s = Surface area of pile shaft in contact with the soil

$\bar{f}_s$ = Average ultimate side resistance per unit area

In general the side resistance will be a function of depth below the surface, because both the undrained strength c_u (short term undrained analysis) and the effective stresses (long term analysis) increase with depth. The average shear stress can be expressed mathematically as

$$\bar{f}_s = \frac{1}{L}\int_0^L f_s \, dz$$

where L is the length of the pile

Total stress analysis (clayey soils)

For these soils the limiting capacity is often controlled by the short term (undrained) condition.

Base resistance

This is a simple bearing capacity problem, that is

$$P_{bu} = A_b \, (f_b + p_o) = A_b \, q_f$$

where q_f is the ultimate bearing capacity. For a soil with $\phi_u = 0$ the ultimate bearing capacity can be written

$$q_f = N_c c_u + \gamma D = N_c c_u + p_o$$

The net ultimate resistance is simply

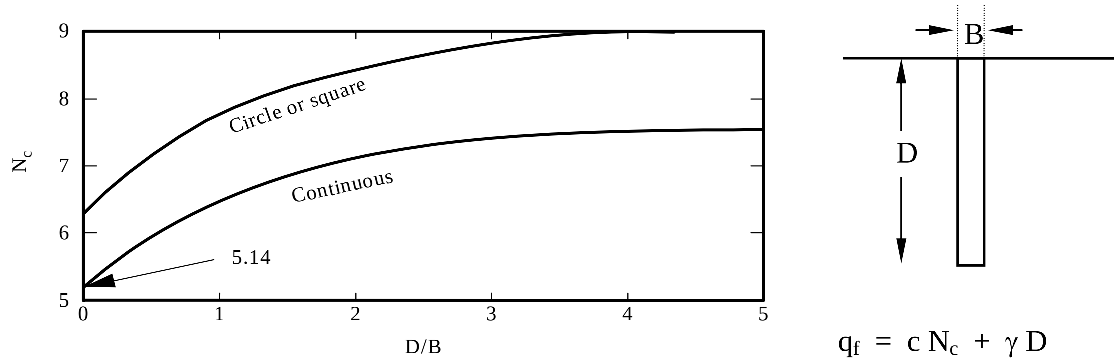
$$f_b = N_c c_u$$

and the ultimate base resistance approximately

$$P_{bu} = A_b (N_c c_u + p_o)$$

It is conventional to take $c_u = c_{ub}$ where c_{ub} is the undrained shear strength of the soil at the pile base and to assume ϕ_u is zero. The value of N_c can then be obtained from Skempton's chart (p28 Data Sheets) which is applicable for $\phi_u = 0$.

When using this chart it is important to check the length to diameter ratio L/D (D/B on the chart). It is normally assumed that pile bases may be treated as deep foundations and that $N_c = 9$. However, if L/D is less than 4, N_c will be lower than 9 as shown in the chart below and the ultimate capacity will be similarly reduced.



Skempton's chart for the Ultimate Bearing Capacity of Clay ($\phi_u = 0$)

Side Resistance

Both total stress and effective stress methods of analysis are used to estimate the side resistance for saturated clays. Here we consider only the total stress or α method.

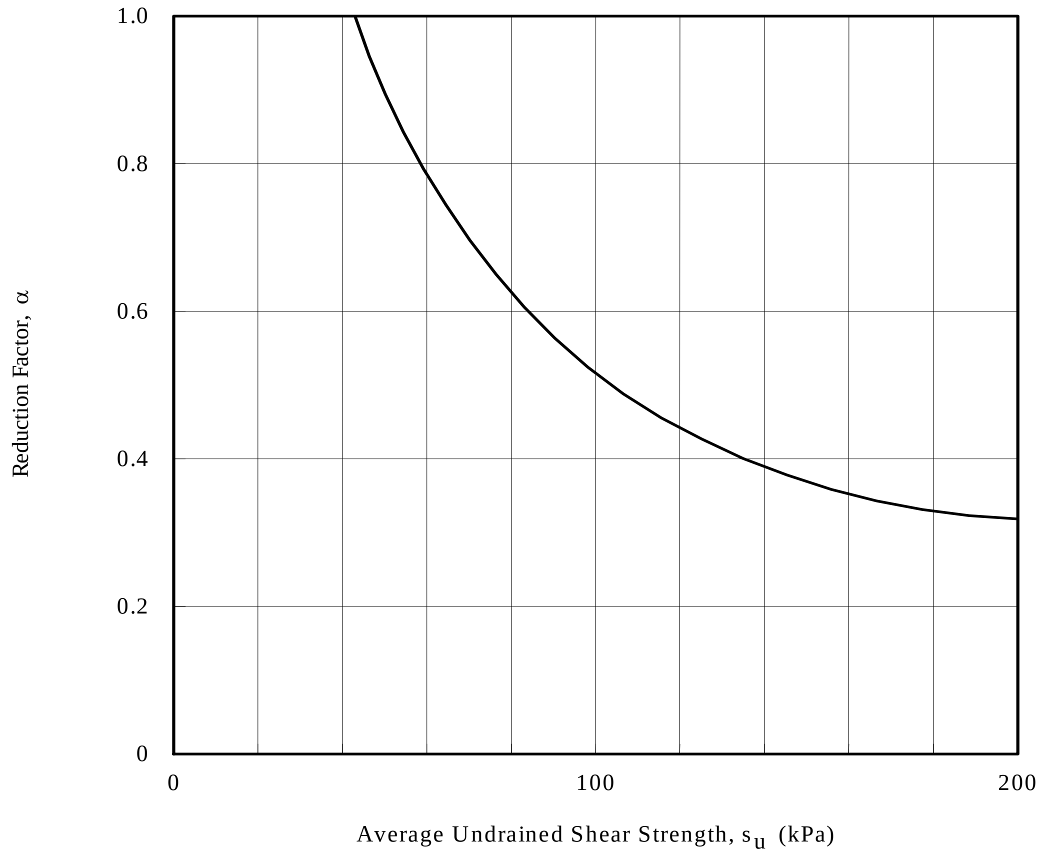
$$f_s(z) = \alpha \, c_u(z)$$

$c_u(z)$ = undrained strength of the soil at depth z

α = an empirical reduction factor which depends on:

- soil type
- pile type
- soil strength (see chart below, p105 Data Sheets)
- method of installation
- time since installation

In the absence of additional information the chart below may be used to estimate α.



REDUCTION FACTOR α vs UNDRAINED SHEAR STRENGTH
FOR PILES IN CLAY

Example

Determine the ultimate axial load of a 10 m long pile, 1 m in diameter where the site consists of a 6 m layer of soft clay with c_u = 25 kPa overlying a deep deposit of stiff clay with c_u= 120 kPa. The soil has γ_{sat} = 18 kN/m^3 and the pile is constructed from reinforced concrete with a unit weight of 22 kN/m^3.

1. Side resistance

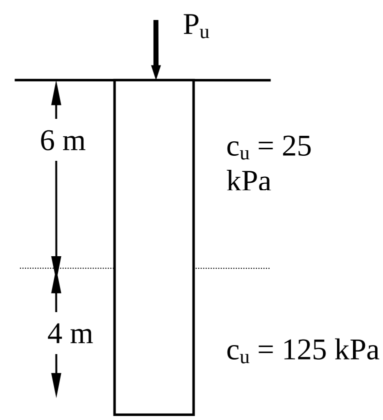
Average shear stress

$$\bar{f}_s = \frac{1}{L}\int_0^L f_s \, dz$$

$$= \frac{1}{10}(\alpha_1 c_{u_1} z_1 + \alpha_2 c_{u_2} z_2)$$

$$= \frac{1}{10}(1 \times 25 \times 6 + 0.43 \times 120 \times 4)$$

$$= 35.6 \text{ kPa}$$



Ultimate shaft resistance

$$P_{su} = \bar{f}_s A_s$$
$$= 35.6 \times \pi \times d \times L$$
$$= 1.12 \text{ MN}$$

2. Base Resistance

$$P_{bu} = A_b (f_b + p_o) = A_b (N_c c_{ub} + p_o)$$

$$L/D = 10 \rightarrow N_c = 9$$

$$f_b = 9 \times 120 = 1080$$
$$p_o = 18 \times 10 = 180$$

$$P_{bu} = \frac{\pi \times 1^2}{4}(1260) = 0.99 \text{ MN}$$

3. Ultimate Axial Capacity

$$P_u = P_{su} + P_{bu} - W$$
$$= 1.12 + 0.99 - \frac{\pi \times 1^2 \times 10 \times 22}{4 \times 1000}$$
$$= 1.94 \text{ MN}$$

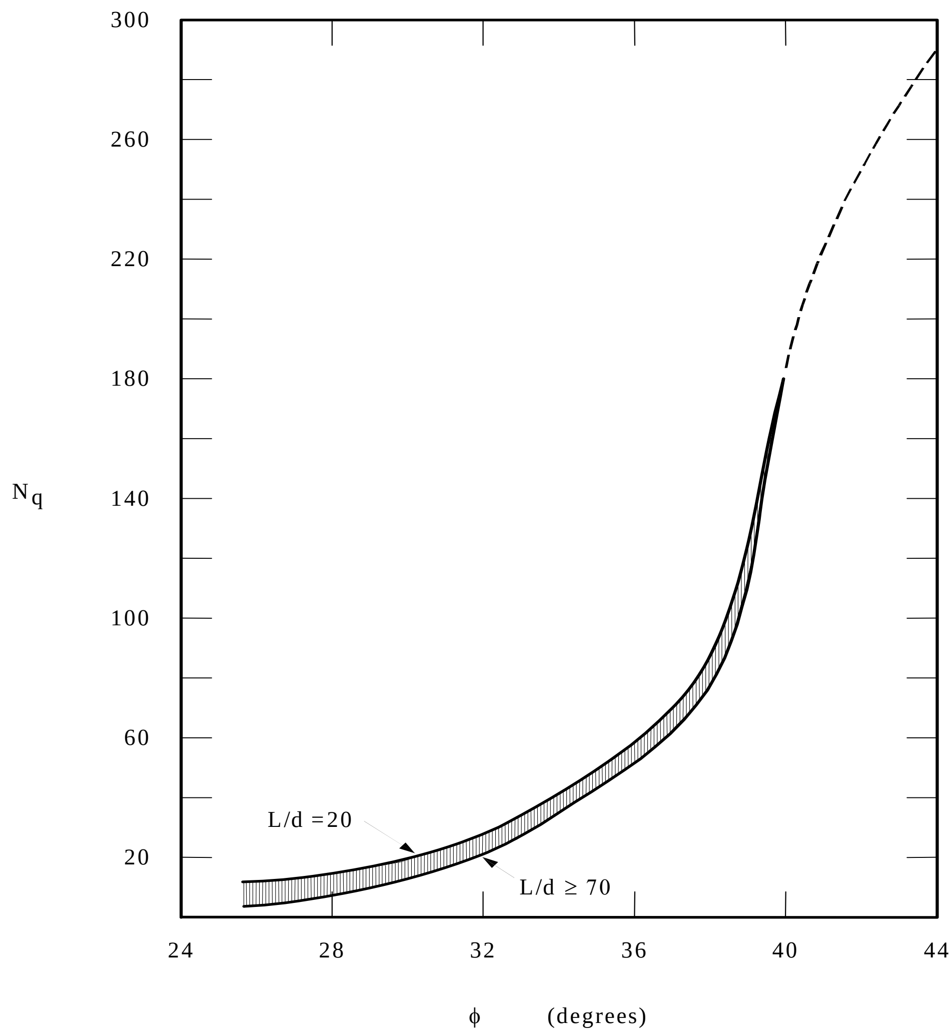
Effective Stress Analysis (sandy soils)

Base resistance

The net end bearing pressure f_b can be expressed in terms of the effective vertical stress at the pile base σ'_v, and a bearing capacity factor N_q

$$f_b = N_q \sigma'_v$$

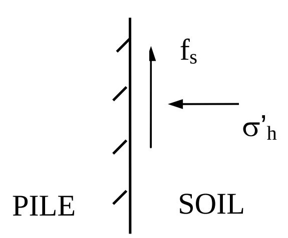
A wide range of values are reported for N_q but the most widely used are those derived by Berezantev et al shown below (p 107 Data Sheets). N_q is very sensitive to the friction angle, particularly for friction angles greater than 36^o. Because of the sensitivity to ϕ' the most sensible value to choose in design is the ultimate (critical state) value, ϕ'_{CS}.



VALUES OF BEARING CAPACITY FACTOR N_q
(After Berezantzev et al, 1961)

Side Resistance

Consider friction between the pile and the soil



$$f_s = \sigma'_h \tan\delta$$

δ = friction angle between the soil and the pile

σ'_h = horizontal effective stress (acting normal to the pile-soil interface). The horizontal stress is difficult to determine and it is normally expressed as:

$$\sigma'_h = K \sigma'_v$$

where K is a factor (earth pressure coefficient) that accounts for the initial stress state and the stress changes due to installation.

For the purpose of performing simple calculations it is convenient to assume that

- $\delta = \phi'_{cs}$
- $K = N_q/50$

The skin friction at any depth can then be calculated from:

$$f_s = \frac{N_q}{50} \sigma'_v \tan \phi'_{cs}$$

and the average is

$$\bar{f}_s = \frac{1}{L} \int_0^L \frac{N_q(z)}{50} \sigma'_v(z) \tan \phi'_{cs}(z)\, dz$$

Note this procedure is quite different from the recommended procedure in the Australian Piling Standard

Ultimate Axial Capacity of Pile Groups

Piled foundations usually consist of many piles. These may be distributed uniformly underneath a structure or in distinct pile groups under heavily loaded areas. In design a check should be made of the ultimate capacity of the group. This will be the lesser of:

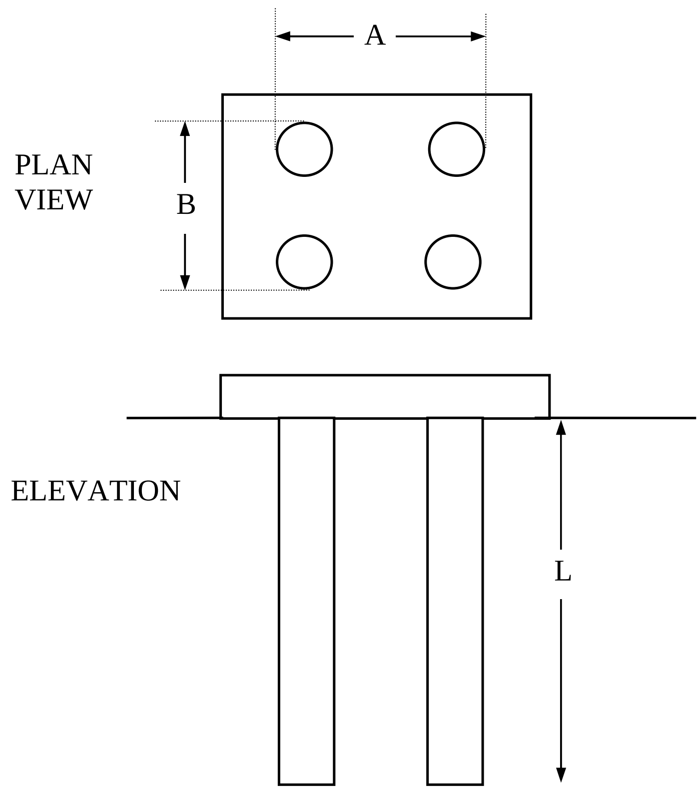
1. The sum of the ultimate loads of the individual piles, that is $n P_{u1}$

   where n = the number of piles in the group

   P_{u1} = The ultimate load for a single pile. (Each pile is assumed to act independently)

2. The ultimate capacity of an equivalent block containing the piles and the soil between the piles

For example consider the four pile group shown below. Both the base and shaft resistance now need to be considered.



Shaft Resistance of the Group P_{su} (group) $= 2 (A + B) L \bar{f}_s$

Base Resistance of the Group P_{bu} (group) $= A B f_b$

The net base resistance, f_b (= $N_c c_{ub}$) is the same as for a single pile, but the value of N_c may change as a result of the different depth to width ratio which becomes L/A rather than L/D for a single pile.

Tutorial Problems - Bearing Capacity

1. A strip footing 3 m wide buried 1.2 m below the surface caries a central vertical load of 15 kN/m of the footing. What is the factor of safety against failure if the load is applied:

   (a) instantaneously

   (b) very slowly

   The soil is a clay with the following properties:

   $c_u = 25$ kN/m^2, $\phi_u = 0$, $c' = 5$ kN/m^2, $\phi' = 20^o$, $\gamma_{sat} = 19$ kN/m^3

   Assume the water table is at the ground surface and use Terzaghi's bearing capacity theory for shallow foundations.

2. Determine the maximum depth that a 4 m by 4 m square temporary excavation with well supported side walls can be dug in a clayey soil with properties $c_u = 15$ kN/m^2, $\phi_u = 0$, $\gamma_{sat} = 18$ kN/m^3.

3. A single pile, 0.8 m in diameter, is to be used in a deep uniform sandy deposit. Calculate the length of pile required if the pile is to carry a load of 1 MN with a factor of safety of 3 against ultimate failure. The sand is dry with $\gamma_{dry} = 16$ kN/m^3 and $\phi'_{cs} = 35^o$. Assume that the weight of the pile is identical to the weight of soil it replaced.

4. A 3 × 3 pile group is to be used to carry a vertical load at a site where the ground conditions of a deep uniform deposit of a clayey soil which has undrained strength $c_u = 50$ kN/m^2. Calculate the ultimate capacity of the group if the piles are 0.5 m in diameter, 8 m long, and are placed with centre to centre spacing of 2 m. Assume that the piles have a similar unit weight to the soil.